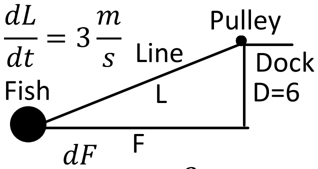
C12 - 4.2 - Pythag/Area/Trig Rel Rates Notes

Fisherperson.

$\frac{dL}{dt} = 3\frac{m}{s}$



$\frac{dF}{dt}_{|F=8} = ?$

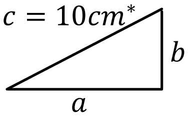
*Substitute Constants!

$$F^2 + D^2 = L^2$$
$$F^2 + 6^2 = L^2$$
$$2F\frac{dF}{dt} + 0 = 2L\frac{dL}{dt}$$
$$2(8)\frac{dF}{dt} + 0 = 2(10)(-3)$$

$F^2 + D^2 = L^2$ $\quad L = 10$

$$\frac{dF}{dt} = -\frac{15}{2}\frac{m}{s}$$

How is the Area Changing, a=6 cm?



$\frac{dA}{dt}\Big|_{a=6\ cm} = ?$

$\frac{da}{dt} = 2\frac{cm}{s}$

$$A = \frac{1}{2}bh$$
$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt}h + \frac{dh}{dt}b\right)$$
$$\frac{dA}{dt} = \frac{1}{2}\big((2)(6) + (-1.5)(8)\big)$$
$$\frac{dA}{dt} = 0\frac{cm^2}{s}$$

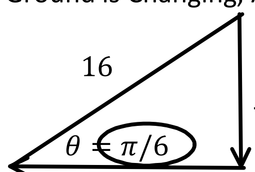
$a^2 + b^2 = c^2$ $\quad b = 8$

$$a^2 + b^2 = c^2$$
$$a^2 + b^2 = 10^2$$
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$
$$2(6)(2) + 2(8)\frac{db}{dt} = 0$$
$$\frac{db}{dt} = -\frac{3}{2}\frac{cm}{s}$$

A 16 ft Ladder slides Down a wall at a Rate of 3 ft/s. Find Rate : Base of Ladder Sliding away from wall, Angle on Ground is Changing, Area is Changing ; when Ladder is at Height of 8 ft on the wall.

$\frac{dA}{dt}\Big|_{y=8} = ?$

$\frac{d\theta}{dt}\Big|_{y=8} = ?$



$x = 8\sqrt{3}$ $\quad \frac{dx}{dt}|_{y=8} = ?$

$y = 8^*$

$\frac{dy}{dt} = -3\frac{ft}{s}$

$$x^2 + y^2 = c^2$$
$$x^2 + y^2 = 16^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(8\sqrt{3})\frac{dx}{dt} + 2(8)(-3) = 0$$
$$\frac{dx}{dt} = \frac{3}{\sqrt{3}} = \sqrt{3}\frac{ft}{s}$$

$$x^2 + y^2 = c^2$$
$$x^2 + 8^2 = 16^2$$
$$x = \sqrt{16^2 - 8^2}$$
$$x = \sqrt{192}$$
$$x = 8\sqrt{3}$$

$$sin\theta = \frac{8}{16}$$
$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\theta = \frac{\pi}{6}$$

*Real life is in Radians.

$$cos\theta = \frac{x}{16}$$
$$-sin\theta\frac{d\theta}{dt} = \frac{1}{16}\frac{dx}{dt}$$
$$-\frac{8}{16}\frac{d\theta}{dt} = \frac{1}{16}\sqrt{3}$$
$$\frac{d\theta}{dt} = -\frac{\sqrt{3}}{8}\frac{rad}{s}$$

Put Constant on the Bottom if can.

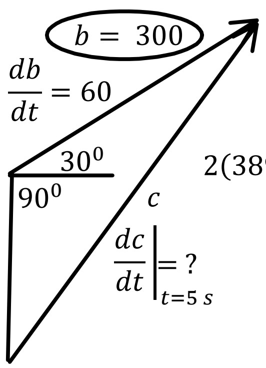
$$A = \frac{bh}{2}$$
$$\frac{dA}{dt} = \frac{1}{2}\left(\sqrt{3}(8) + (-3)(8\sqrt{3})\right)$$
$$\frac{dA}{dt} = -\frac{16\sqrt{3}}{2}$$
$$\frac{dA}{dt} = -8\sqrt{3}\frac{ft^2}{s}$$

A float plane rising at 30 degrees above the horizontal flies over a boat at an altitude of 100 m at 60 m/s. How fast is the distance between the boat and the plane increasing after five seconds?

$$d = vt$$
$$d = 60 \times 5$$
$$d = 300$$

$b = 300$

$\frac{db}{dt} = 60$



$a = 100$

$\frac{dc}{dt}\Big|_{t=5\ s} = ?$

$$c^2 = a^2 + b^2 - 2abcosC$$
$$2c\frac{dc}{dt} = 0 + 2b\frac{db}{dt} - 2acosC\frac{db}{dt}$$
$$2(389.8)\frac{dc}{dt} = 0 + 2(300)(60) - 2(100)\left(-\frac{\sqrt{3}}{2}\right)(60)$$
$$\frac{dc}{dt} = 59.5\frac{m}{s}$$

Cosine Law

$$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$
$$120^0 = \frac{7\pi}{6}$$

$$c^2 = a^2 + b^2 - 2abcosC$$
$$c = \sqrt{100^2 + 300^2 - 2(100)(300)\cos\frac{7\pi}{6}}$$
$$c = 389.8\ m$$